ONE PULSED HYDRODYNAMIC MODEL OF A CRATERING EXPLOSION

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It is well-known that in an explosion, in a number of cases, the compressibility of the medium, its stability and plastic properties, and frictional forces can be neglected in comparison with the inertial forces. Then the model of an ideally incompressible liquid is obtained, for which the equations of motion in the pulsed formulation of the problem of hydrodynamics have the form

$$\mathbf{v} = -(1/\rho)\nabla p; \tag{1}$$

$$\operatorname{div} \mathbf{v} = 0, \tag{2}$$

where $\mathbf{v}$ is the velocity vector; ρ is the density of the medium; and p is the pulse pressure. It follows from relation (1) that $|\nabla p|$ and the magnitude of the velocity $v = |\mathbf{v}|$ are connected by a linear relation. This model is used for solving problems of the determination of the dimensions of ground throwout craters or zones of crushing in rocks (see, for example, [1-7]). In this case, the given model is of somewhat different form [5]. According to the liquid model [1] (we shall call it model 1), the medium is considered as an ideal incompressible liquid in the whole region occupied by it, so that in all this region the linear relation between v and ∇p is valid. It follows from the latter that with any suitably small values of $|\nabla p| > 0$, the whole medium acquires an instantaneous velocity field. But since the region of action of an explosion is limited, the concept of critical velocity v_* is introduced, during the attainment of which fracture of the given medium occurs [6]. The dimensions of the throwout crater are determined from the condition that the critical value of the velocity is reached at the edge of this excavation.

In the solid-liquid model [2, 4] (we shall call it model 2), the medium is described by Eqs. (1) and (2) only in the region where $v > v_*$. Outside of this region, the medium is assumed to be an absolutely solid body. According to this model, the shape of the throwout crater is found to be a line of flow, along which $v = v_*$. In model 2, the linear dependence between v and ∇p occurs only when $v > v_*$ (when $v < v_*$, it is not possible to say anything about the relation between $|\nabla p|$ and v), and the instantaneous velocity field originating during the explosion contains only the velocities $v \geq v_*$. In Fig. 1, model 1 corresponds to the straight line 0*a*b and model 2 corresponds to the straight line *a*b.

We note that in the description of model 2 we can speak more conveniently, not of the attainment of the value v_*, but about a certain modulus (absolute value) of the gradient of the pulsed pressure λ, during the attainment of which motion begins with a velocity v_* (see Fig. 1). We shall call the quantity λ the initial gradient, by analogy with the problems of nonlinear filtration theory (see, for example, [8-11]). In terms of this theory, model 2 corresponds to the so-called piecewise-explosive law of filtration [12].

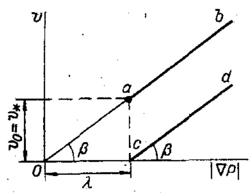

Fig. 1

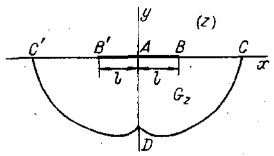

Fig. 2

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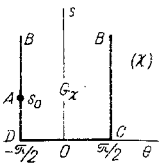

Fig. 3

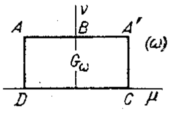

Fig. 4

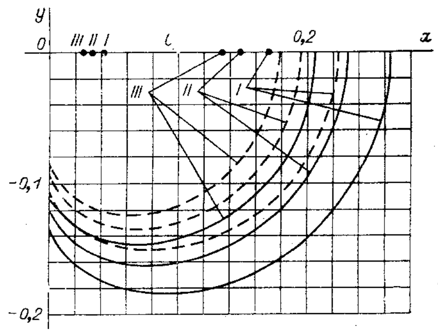

Fig. 5

TABLE 1

Alternative No.	v'_A	l'	Model 1	Model 2		Model 3	
			x'_C	x'_C	$-y'_D$	x'_C	$-y'_D$
I	10	0,0427	0,1703	0,2217	0,1018	0,2630	0,1310
II	14	0,0332	0,1491	0,1952	0,0914	0,2292	0,1164
III	18	0,0276	0,1354	0,1778	0,0842	0,2057	0,1055

In addition to the models considered, we shall suggest a model 3 (the question of the suitability of application of one or the other model for calculating the effect of an explosion can be resolved only on the basis of experimental data). The instantaneous velocity field, just as in model 2, originates with the attainment $|\nabla p|$ of the initial gradient λ; but in contrast from model 2, this field contains the velocities $v \geq 0$ and not only $v \geq v_*$. The law of motion corresponding to this model is shown in Fig. 1 by the straight line cd (it is supposed that the uniform incompressible medium, characterized by the initial gradient λ, is one and the same, i.e., $\tan \beta = 1/\rho$, ρ =const). The equation of motion is represented conveniently in the form

$$\nabla p = -(\lambda + \rho v)(\mathrm{v}/v), \ \lambda \leqslant |\nabla p| < \infty, \ \operatorname{div} \mathrm{v} = 0. \tag{3}$$

The boundary of the excavation is found from the condition of equating to zero the magnitudes of the velocity on this line. We note that if during the explosion two zones are distinguished – a zone of fracture of the medium and a throwout zone – as is done, for example, in [13, 14], then probably model 3 corresponds more to the problem of determining the boundary of the crater of fracturing of the medium.

We shall consider the problem concerning the explosion of a plane cord charge on the surface of homogeneous ground. This problem is solved by model 2 in [4]. We shall simplify the solution of this problem using model 3. In order that the dimensions of the craters obtained by the different models can be compared, we shall assume that the density of the medium ρ, the pulsed pressure at the charge p_0, the initial gradient of the pulsed pressure λ, and the width $2l$ of the plane cord charge are identical.

Suppose that during the explosion of a plane cord charge with cross section BAB′ and width $2l$ an excavation CDC′ has been formed (Fig. 2). In view of symmetry, we shall consider only the right-hand half of the region of motion, which we shall denote by G_z, its boundary being denoted by Γ_z $(z = x + iy)$. Knowing the quantities p_0, ρ, λ, and l, and based on model 3, we shall construct the section CD of the boundary Γ_z.

We introduce the magnitude of the velocity v_0 by the relation $v_0 = \lambda/\rho$. If we take into account that $\tan \beta = 1/\rho$, we obtain $v_0 = v_*$, i.e., the quantity v_0 will be the critical velocity v_* for model 2 (see Fig. 1). The system of equations (2) and (3) can be written in the form

$$\partial\varphi/\partial x = (1 + v_0/v)v_x, \ \partial\varphi/\partial y = (1 + v_0/v)v_y, \ \partial v_x/\partial x + \partial v_y/\partial y = 0, \tag{4}$$

where $\varphi = -p/\rho$ and v_x, v_y are the projections of the velocity on the coordinate axes.

The corresponding boundary-value problem thus becomes as follows: to construct the section CD of the boundary Γ_z of the region G_z and to determine in this region the functions $\varphi(x, y)$, $v_x(x, y)$ and $v_y(x, y)$ satisfying the nonlinear system of equations (4), according to the boundary conditions

$$\begin{gathered}\varphi = -\varphi_0 \text{ on } AB, \ \varphi = 0 \text{ on } BC, \\ v_x = 0 \text{ on } AD, \ v_x = v_y = 0 \text{ on } CD,\end{gathered} \tag{5}$$

where $\varphi_0 = p_0/\rho$.

We introduce the stream function ψ by the relations

$$\partial\psi/\partial x = -v \sin \theta,\ \partial\psi/\partial y = v \cos \theta,$$

where θ is the angle of inclination of the velocity vector to the axis x. Then system (4), following [15, 16], can be transformed to the form

$$\frac{\partial\varphi}{\partial\theta} = \frac{1}{\varkappa(s)}\frac{\partial\psi}{\partial s},\ \frac{\partial\varphi}{\partial s} = -\frac{1}{\varkappa(s)}\frac{\partial\psi}{\partial\theta}, \tag{6}$$

where

$$s = 2\ln\left(\sqrt{1+\frac{v}{v_0}} + \sqrt{\frac{v}{v_0}}\right),\ \varkappa = \left(1+\frac{v_0}{v}\right)^{-3/2}, \tag{7}$$

whereby we choose $s = 0$ when $v = 0$. We note that on the line $s = 0$, Eqs. (6) are degenerate. Conversion to the physical region is effected by the formula

$$dz = dx + idy = (e^{i\theta}/v)(vd\varphi/(v_0 + v) + id\psi). \tag{8}$$

We introduce the complex function $w(\chi) = \varphi(\theta, s) + i\psi(\theta, s)$, where $\chi = \theta + is$. In the χ plane of the region G_z, according to the principle of the argument (see, for example, [10]), there corresponds a half-zone G_χ (Fig. 3). Taking account of Eq. (5), for the construction in G_χ of the bounded and continuous function $w(\chi)$, we have the following boundary conditions:

$$\varphi = -\varphi_0 \text{ on } AB,\ \varphi = 0 \text{ on } BC,\ \psi = 0 \text{ on } ADC. \tag{9}$$

Later, we shall solve the problem in semiinverse formulation: the velocity v_A at the point A will be specified, and the half-width of the charge l and the corresponding excavation will be found.

Knowing v_A, we find $s_A = s_0$ by the first of formulas (7). We map conformally the region G_χ on the half-plane $\operatorname{Im}\zeta > 0$ by the function

$$\zeta = \sin \chi. \tag{10}$$

As a result of this, the point $A(\chi_A = -\pi/2 + is_0)$ transfers to a point on the ξ axis with abscissa $\xi_A = -\cosh s_0$ (we denote $\cosh s_0 = 1/k$). We map the half-plane $\operatorname{Im}\zeta > 0$ conformally on the rectangle G_ω ($\omega = \mu + i\nu$) with sides $AA' = DC = 2K(k)$ and $AD = A'C = K(k')$ by the elliptical integral of the first species:

$$\omega = F(\arcsin \zeta,\ k), \tag{11}$$

where K is a total elliptical integral of the first species; $k' = \sqrt{1-k^2}$ (Fig. 4). Then in order to find the function $w(\omega)$ in the region G_ω, we arrive at the solution of the equation

$$\partial\varphi/\partial\mu = (1/\tilde{\varkappa})\partial\psi/\partial\nu,\ \partial\varphi/\partial\nu = -(1/\tilde{\varkappa})\ \partial\psi/\partial\mu \tag{12}$$

with the boundary conditions (9), where, taking into account Eqs. (10) and (11),

$$\tilde{\varkappa}(\mu\nu) = \varkappa\{s[\xi(\mu,\ \nu),\ \eta(\mu,\ \nu)]\}.$$

Solving system (12) relative to the function $\varphi(\mu, \nu)$, we obtain for its determination the equation

$$\frac{\partial}{\partial\mu}\left(\tilde{\varkappa}\frac{\partial\varphi}{\partial\mu}\right) + \frac{\partial}{\partial\nu}\left(\tilde{\varkappa}\frac{\partial\varphi}{\partial\nu}\right) = 0 \tag{13}$$

and the boundary conditions

$$\varphi = -\varphi_0 \text{ on } AB,\ \varphi = 0 \text{ on } BC,\ \partial\varphi/\partial\mu = 0 \text{ on } AD; \tag{14}$$

$$\lim_{\nu\to 0}\left(\tilde{\varkappa}\frac{\partial\varphi}{\partial\nu}\right) = 0 \text{ on } CD. \tag{15}$$

The solution of the problem (13)-(15) is found by a numerical method developed, for example, in [17]. We note that condition (15), as follows from the results of [18], can be substituted in the problem being considered by the condition $\partial\varphi/\partial\nu = 0$ on CD.

Having determined the function $\varphi(\mu, \nu)$ and, consequently [taking into account Eqs. (10) and (11)], also the function $\varphi(\theta, s)$, by the conversion formula (8) we find the equation of the required section CD of the boundary Γ_z:

$$z = \frac{1}{v_0} \int\limits_{-\pi/2}^{\theta} e^{i\theta} \frac{\partial\varphi}{\partial\theta}\Big|_{s=0} d\theta + iA^*, \ (-\pi/2 \leqslant \theta \leqslant \pi/2), \tag{16}$$

where

$$A^* = -\frac{1}{v_0} \int\limits_{-\pi/2}^{\pi/2} \frac{\partial\varphi}{\partial\theta}\Big|_{s=0} \sin\theta d\theta.$$

In order to determine the half-width of the charge l, we have

$$l = \int\limits_{s_0}^{\infty} \frac{\varkappa}{v} \frac{\partial\varphi}{\partial\theta}\Big|_{\theta=-\pi/2} ds. \tag{17}$$

Formulas (16) and (17) give the solution of the problem.

As applicable to model 2, the proposed scheme for constructing the solution is simplified and allows an analytical solution of the problem in the direct formulation to be found quite simply. We shall derive this solution in a form somewhat different from [4] in order to compare both models.

We introduce

$$\tilde{\chi} = \theta + i\tilde{s} = i \ln [(1/v_0) dw/dz], \tag{18}$$

where $\theta = \arg \bar{v}$ and $\tilde{s} = \ln(v/v_0)$. The region $G_{\tilde{\chi}}$, corresponding to the physical region G_z, has the same form as G_χ (see Fig. 3), but $\tilde{s}_A = \tilde{s}_0 = \ln(v_A/v_0)$. From system (6), which assumes the form

$$\partial\varphi/\partial\theta = \partial\psi/\partial\tilde{s}, \ \partial\varphi/\partial\tilde{s} = -\partial\psi/\partial\theta, \tag{19}$$

it follows that the function $w(\tilde{\chi}) = \varphi(\theta, \tilde{s}) + i\psi(\theta, \tilde{s})$ is analytic in the region $G_{\tilde{\chi}}$. Solving in this region the mixed boundary-value problem (9) and (19), we determine

$$w(\tilde{\chi}) = \frac{i}{\pi} \ln \frac{\alpha - 1 + 2\sin\tilde{\chi} + 2\sqrt{(\sin\tilde{\chi} - 1)(\sin\tilde{\chi} + \alpha)}}{\alpha + 1}, \tag{20}$$

where $\alpha = \cosh \tilde{s}_0$. Using Eqs. (18) and (20), after evaluation of the integrals we obtain

$$z = \frac{1}{\pi v_0} \left\{ \sqrt{\sin\tilde{\chi} + \alpha} \left(\sqrt{\sin\tilde{\chi} + 1} - \sqrt{\sin\tilde{\chi} - 1} \right) + \frac{\alpha - 1}{2} \ln \frac{2\sqrt{(\sin\tilde{\chi} - 1)(\sin\tilde{\chi} + \alpha)} + 2\sin\tilde{\chi} + \alpha - 1}{2\sqrt{(\sin\tilde{\chi} + 1)(\sin\tilde{\chi} + \alpha)} + 2\sin\tilde{\chi} + \alpha + 1} \right. + B^*. \tag{21}$$

From the corresponding points A and B in the planes z and $\tilde{\chi}$, we find the constant $B^* = l - 1/\pi v_0$ and the equation for determining the parameter α:

$$[(\alpha - 1)/2] \ln [(\alpha + 1)/(\alpha - 1)] = 1 - \pi v_0 l.$$

Knowing α and B^*, and assuming on the section DC of the boundary of the region $G_{\tilde{\chi}}$ the value $\tilde{\chi} = \theta (-\pi/2 \leqslant \theta \leqslant \pi/2)$ and factoring out in Eq. (21) $\mathrm{Re}\, z = x(\theta)$ and $\mathrm{Im}\, z = y(\theta)$, we obtain a parametric equation for the required boundary of the ground throwout crater.

Finally, we note that in the case of model 1, i.e., the liquid model,

$$w = (\varphi_0/\pi) \arcsin [(z^2 + l^2)/(z^2 - l^2)] - \varphi_0/2,$$

whence, in order to determine the edge of the throwout crater x_C, we have

$$x_C = (l^2 + 2\varphi_0 l/\pi v_0)^{1/2}. \tag{22}$$

Taking into account that when carrying out the numerical calculations and comparing the excavations obtained by the different models it is more advantageous to use scaled quantities, we introduce them in the following way:

$$z' = zv_0/\varphi_0, \ w' = w/\varphi_0, \ v' = v/v_0.$$

Then

$$v_0' = 1, \ \varphi_0' = 1,$$

and

$$l' = lv_0/\varphi_0. \tag{23}$$

Thus, when solving the problem in scaled quantities by model 3, $\varphi'_0=1$, $v'_0=1$, and $v'_A > 0$ are specified in advance. During this solution, the boundary of the excavation and the half-width of the charge l' as a function of the value of v'_A are determined. In order to convert from this solution to the solution in a dimensional physical plane, of the values of the parameters l, v_0, and φ_0 usually known in advance (in the direct formulation), only two should be specified, since the third will be determined by formula (23).

Table 1 gives the values of l' for three alternatives of the given data v'_A found by Eq. (17) and the abscissas x'_C corresponding to them and found by Eqs. (16), (21), and (22), for the three models considered, and the ordinates y'_D for models 2 and 3. In Fig. 5, the boundaries of the crater are constructed and the sizes of the charges corresponding to them are shown (the solid line corresponds to model 3; the dashed line corresponds to model 2). The edges of the excavation, found by model 1, are denoted by dots.

It follows from the results of the calculations, Table 1, and Fig. 5 that the excavation produced is greater according to model 3 than according to model 2 (in area, as well as in depth and width). This can be explained physically in the following way. With one and the same initial gradient of the pulsed pressure λ, according to model 2 the explosion energy is taken into account only for $v \geq v_* > 0$, while in model 3 all the energy is taken into account, starting with $v \geq 0$. The explanation as to why according to model 1 the edge of the excavation is located closer to the charge than according to model 2 is given in [5].

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PLANE WAVES IN NONLINEAR VISCOUS MULTICOMPONENT MEDIA

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Wave processes in multicomponent media (liquid and water-saturated soil with bubbles of gas, suspensions, etc.) have been studied in [1-20] and other investigations.

In [1] it was assumed that the space is filled with a number of continuous media, each of which corresponds to a component of the medium. The investigation was concerned with interpenetrating motions of these media (in the general case each moves with its own velocity and pressure). In the model of [2] the multicomponent medium was regarded as a homogeneous continuous medium with a compressibility equation taking account of the compressibility and the presence of components that were in an equilibrium state. In [3] the multicomponent medium was regarded as homogeneous, and the compressibility of the gaseous component was determined by Hugoniot's adiabatic curve. The reflection of a plane wave from a solid partition for various angles of incidence was investigated in [4] on the basis of [2], using electronic computers. The problem of the propagation of a wave produced by the explosion of the spherical charge of a blast wave, using the model of [2] as a basis, was solved by means of electronic computers in [5]. The authors of [6] proposed a model of a homogeneous medium analogous to that of [2] and obtained solutions of problems concerning the passage of a wave through a layer of water with gas bubbles and the reflection of the wave from a fixed boundary. The special characteristics of the structure of waves in water with gas bubbles and the effect of viscosity dissipation related to the motion of the bubbles with respect to the liquid were considered in [7]. In the model of [8] the pulsation of the bubbles was assumed to conform to Lamb's equation, i.e., the lack of equilibrium between the phases was taken into consideration. The case of strong shock waves, on the basis of [8], was considered in [9]. In [10, 11] it was shown that in a liquid with gas bubbles, for specific relationships between the viscosity, the load, and the bubble radius, there is formed a wave with an oscillator structure. In [12] the structure of a wave was investigated on the basis of the model of [13], with oscillations taken into consideration. Equations for the mechanics of a two-velocity two-temperature medium with two pressures were proposed in [14]. In [15], on the basis of [14], the structure of a stationary wave was investigated with thermal conductivity taken into account. It was shown that the nature of the pulsation depends substantially on the heat exchange between the phases. It was noted that the experiments of [11] should be analyzed with the time-dependent change of structure taken into account. In the experiments of [16] it was established that an increase in the intensity of the wave leads to an increase in the frequency and amplitude of the oscillations on the front, while an increase in the bubble diameter leads to a decrease of the frequency and an increase of the amplitude. Weak waves were considered. The authors of [17] obtained numerical solutions making it possible to determine the amplitude oscillations on the wave front, the velocity of propagation of the wave, and the time required for establishing a stationary structure. Waves in water-saturated rocks were considered in [18]. The authors of [18] obtained an equation describing weak longitudinal waves with inertial relaxation taken into account. The effect of the tension surface was investigated in [19]. In [20] the model of [2] was improved by the introduction of nonlinear diagrams for the dynamic and static compression of the multicomponent medium, making it possible to introduce bulk viscosity. The effect of viscosity was considered in a somewhat different manner in [21].

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